

sure p_{10} that was determined from prior experiments with circular orifices and the theory of Potter et al. The data from the circular orifices agree with the previous data from circular orifices, with the semiempirical-theoretical line splitting the small differences in measurements.

Also shown are the data from the oblong orifices, using an equivalent diameter d_{eq} to see if better correlation resulted. The equivalent diameter was determined by equating the orifice area to the area of a circle with an equivalent diameter, i.e.,

$$A_{\text{orifice}} = (\pi/4) d_{eq}^2$$

Although the results are not entirely conclusive, this method of handling the data does not appear to be as good as simply using the streamwise dimension b as the effective diameter.

Reference

¹ Potter, J. L., Kinslow, M., and Boylan, D. E., "An influence of the orifice on measured pressures in rarefied flow," *Fourth International Symposium on Rarefied Gas Dynamics*, (Academic Press, New York, to be published).

Unsteady Laminar Jet Flame at Large Frequencies of Oscillation

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Nomenclature

a	= characteristic length in problem, jet width
c	= speed of sound
D_{12}	= binary diffusion coefficient
F	= steady-state stream function
$H, S, V,$ Z, U, W	= high frequency representations of solution for $P, \sigma,$ and \mathcal{Y}_K
$h, s, v,$ z, u, w	= high frequency expansion functions for $H, S, V,$ $Z, U,$ and W
i	= complex number, $(-1)^{1/2}$
j	= stoichiometric mass ratio of fuel to oxidizer
m	= total mass flux per unit flame length/ 2π in r direction
M	= Mach number
\mathfrak{M}	= space-dependent part of m perturbation
p	= pressure
P	= space-dependent part of ψ perturbation
q	= stoichiometric heat of reaction nondimensionalized by $\bar{c}_p^* T_\infty^*$
r	= radial coordinate
R_f	= perturbation magnitude of flame movement
Re	= Reynolds number
s	= axial coordinate
t	= time
T	= temperature
$u, (v)$	= velocity of center of mass of a fluid element in the s direction (r direction)
$x = s/Re$	= axial coordinate
Y_K	= mass fraction of K th species
\mathcal{Y}_K	= space-dependent part of Y_K perturbation
α	= $1/(i\omega)^{1/2}$
β	= high frequency variable
γ	= ratio of specific heats
δ	= ± 1
ϵ	= perturbation parameter
μ	= viscosity
$\hat{\mu}$	= boundary-layer variable

ξ	= axial variable
ρ	= density
σ	= space-dependent part of T perturbation
$\tau = t/Re$	= time
ψ	= stream function
ω	= frequency

Subscripts

c	= core centerline value
f	= quantity evaluated at flame
F	= fuel or fuel side of flame
o	= oxidizer or oxidizer side of flame
∞	= freestream values

Superscripts

—	= steady-state quantity
*	= dimensional quantity
(i)	= i th term in expansion in a small parameter

INFORMATION concerning time-dependent combustion processes is very meager but greatly needed. Of several types of unsteadiness or time dependence, the most interesting from the point of view of unstable processes is the periodic phenomenon caused by the action of a periodic sound wave on an otherwise steady-state burning configuration. It appears that an interesting system for study is the laminar jet flame or overventilated diffusion flame. This system may have application to the wake of a burning droplet or the coaxial jet injector used in rocket engines. The occurrence of sound waves acting on such configurations is well known through the knowledge gained on the phenomenon of combustion instability.

High frequency of an imposed sound wave will be considered. "High" is meant in the sense that the cycle time is short compared to a particle transit time of the flame length. This is equivalent to the statement that the cycle time is short compared to a diffusion time transverse to the mixing region.

A method for generating an asymptotic solution valid in the entire flow field is developed. This method for the compressible, reacting, viscous jet may be specialized for simpler cases; as such, the method is general and contributes to unsteady boundary-layer theory. The primary interest in this note from the standpoint of application is to extract information concerning the behavior of the burning rate under the action of the sound wave. A complication in unsteady boundary-layer theory, therefore, enters: that of a moving boundary, the flame. The procedure for treating this is indicated. It should be made very clear that the interest here is in a forced oscillation, at the space and timewise frequency of a sound wave, which is compatible with the boundary-layer equations. This has no bearing on other possible disturbance types, e.g., an intrinsic flow instability consisting of disturbances traveling with the velocity of the fluid rather than at the speed of sound. It is well known that the type treated is most important in determining combustion energy feedback to an acoustic wave. The frequencies of intrinsic oscillations are, in general, not compatible with acoustic frequencies, and no organized interaction occurs, at least for small amplitudes.

Consider the laminar jet flame composed of a fuel rich gas ejected from a pipe into a flowing freestream containing an oxidizer as in Fig. 1. The jet velocity profile is characterized by a velocity u_c^* and the freestream by a velocity u_∞^* . It is assumed that $|u_\infty^* - u_c^*|$ is sufficiently large that $Re = \rho_\infty^* |u_\infty^* - u_c^*| a^* / \mu_\infty^* \gg 1$. At the same time, the respective Mach numbers $M_\infty = u_\infty^* / c_\infty^*$ and $M_c = u_c^* / c_c^*$ are low, such that their squares are negligible compared to unity. The jet is characterized by $Y_{F,c}$ and $T_{c,*}$ and the freestream by $Y_{O,\infty}$ and $T_{\infty,*}$. Flame ignition is assumed instantaneous such that the flame is initiated at the lip of the jet. The fuel diffuses toward the flame from the core, and the oxidizer

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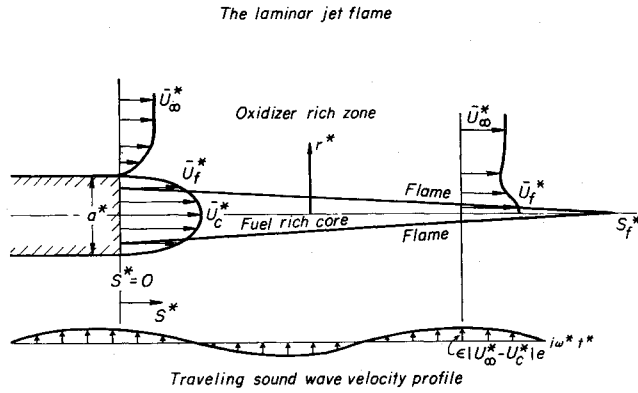


Fig. 1 Model of overventilated laminar jet diffusion flame with a traveling sound wave introduced parallel to the freestream.

diffuses from the surroundings; diffusion occurs, since oxidizer and fuel are being consumed at the flame, and concentration gradients occur.

The velocity profiles will be smoothed by viscosity, and the core will heat up in axial distance because of heat generation at the flame. At some axial length, the fuel concentration will be zero and the flame will then cease to exist, this being the flame length. Infinite reaction-rate kinetics are assumed such that the flame thickness is negligible and the flame becomes a sheet heat source. Reaction at the flame is thus assumed instantaneous and complete. If it is instantaneous, the mass fractions of fuel and oxidizer must be zero at the flame; if it is complete, there must exist a mass feeding rate of oxidizer and fuel to the flame in stoichiometric proportions. Only a binary mixture of gases is considered such that, on the fuel side of the flame, there are only fuel and a fictitious single-component-product gas (or inert), and on the oxidizer side there are only oxidizer and inert. The local burning rate is defined by the mass of fuel being consumed per unit flame length at any axial location per unit time.

In addition to the preceding, the following assumptions will be made: 1) the fluid is a continuum; 2) there is heat transfer by conduction only; 3) there are no over-all mass sources; 4) the flow is laminar; 5) there are no over-all or species body forces; 6) each gas component obeys the perfect gas law, each constituent being thermally and calorically perfect; 7) the specific heats of each species will be the same, implying equal molecular weights; 8) made plausible by 7, mass diffusion occurs by a mass fraction gradient only, not by pressure or thermal diffusion; 9) the viscosities, $\rho^* D_{12}^*$ product, and thermal conductivity will be composition- and pressure-independent and proportional to the first power of temperature; 10) the Prandtl and Schmidt numbers are unity. Nondimensionalize velocities by $|u_\infty^* - u_c^*|$, viscosities by μ_∞^* , stress by $\rho_\infty^* |u_\infty^* - u_c^*|^2$, length by a^* , temperature by T_∞^* , density by ρ_∞^* , time by $a^*/|u_\infty^* - u_c^*|$. Then the unsteadiness is introduced by a traveling isentropic sound wave in the freestream and is periodic in space and time. The velocity perturbation is characterized by magnitude $\epsilon |u_\infty^* - u_c^*|$ with $\epsilon \ll 1$. The form of the velocity in the freestream is

$$u = \bar{u}_\infty + \epsilon \exp\{i\omega[t + (\delta M_\infty s / \bar{u}_\infty)]\} + \mathcal{O}[\epsilon M_\infty^2] + \mathcal{O}[\epsilon^2] \quad (1)$$

$\delta = \pm 1$ depending on the direction of wave travel. The pressure in the freestream is given by

$$p = \frac{\bar{u}_\infty^2}{\gamma M_\infty^2} - \epsilon \left\{ \frac{\delta \bar{u}_\infty}{M_\infty} \exp\left[i\omega\left(t + \frac{\delta M_\infty}{u_\infty} s\right)\right] + \mathcal{O}(1) \right\} \quad (2)$$

The associated state variables are easily constructed. A

homocompositional field is assumed. Solutions of the same periodicity are desired to the burning and mixing region behavior.

The preceding assumptions and restrictions imply the boundary-layer equations with a flame length of $\mathcal{O}[a^* Re]$. This may be deduced by a generalization of Fay's result.¹ The steady-state solution will be assumed known, being available from several known methods, e.g., Refs. 1 and 2.

The unsteady solution is generated by a power series expansion in the small parameter ϵ , and only the first power in ϵ is considered, resulting in a linearized theory. The first order in ϵ , time-dependent boundary-layer equations are extracted after introducing the following boundary-layer variables and the stream function:

$$\psi_r = \rho u r \quad \psi_x + \bar{\mu}_r = -\rho v r$$

$$\bar{\mu} = \int_0^r \rho r' dr' \quad t = \tau \quad \xi = x \quad \omega = \omega Re \quad (3)$$

The solution is assumed in the form

$$\psi = F(\bar{\mu}, \xi) + \epsilon P(\bar{\mu}, \xi) \exp\left[i\omega\left(t + \frac{\delta M_\infty}{\bar{u}_\infty} \xi\right)\right]$$

$$T = \bar{T}(\bar{\mu}, \xi) + \epsilon \sigma(\bar{\mu}, \xi) \exp\left[i\omega\left(t + \frac{\delta M_\infty}{\bar{u}_\infty} \xi\right)\right]$$

$$Y_K = \bar{Y}_K(\bar{\mu}, \xi) + \epsilon Y_{K\epsilon}(\bar{\mu}, \xi) \exp\left[i\omega\left(t + \frac{\delta M_\infty}{\bar{u}_\infty} \xi\right)\right]$$

Then with $\alpha = (i\omega)^{-1/2}$, the unsteady boundary-layer equations to first order in ϵ are

$$P_{\bar{\mu}} \left(1 + \frac{\delta M_\infty}{u_\infty} F_{\bar{\mu}}\right) - \frac{\delta M_\infty}{u_\infty} F_{\bar{\mu}\bar{\mu}} P = \left(1 + \frac{\delta M_\infty}{u_\infty}\right) \bar{T} + \alpha^2 [(r^2 P_{\bar{\mu}\bar{\mu}})_{\bar{\mu}} + P_{\bar{\mu}\bar{\mu}} F_\xi - P_{\bar{\mu}} F_{\bar{\mu}\xi} - P_{\bar{\mu}\xi} F_{\bar{\mu}} + F_{\bar{\mu}\bar{\mu}} P_\xi - \frac{\gamma \delta M_\infty}{u_\infty} (r^2 F_{\bar{\mu}\bar{\mu}})_{\bar{\mu}}] \quad (4a)$$

$$\sigma \left(1 + \frac{\delta M_\infty}{u_\infty} F_{\bar{\mu}}\right) + \frac{\delta M_\infty}{u_\infty} (\gamma - 1) \bar{T} = \bar{T}_{\bar{\mu}} \frac{\delta M_\infty}{u_\infty} P + \alpha^2 [\bar{T}_{\bar{\mu}} P_\xi - P_{\bar{\mu}} \bar{T}_\xi + (r^2 \sigma_{\bar{\mu}})_{\bar{\mu}} + F_\xi \sigma_{\bar{\mu}} - \sigma_\xi F_{\bar{\mu}} - \gamma \frac{\delta M_\infty}{u_\infty} (r^2 \bar{T}_{\bar{\mu}})_{\bar{\mu}}] \quad (4b)$$

$$Y_{K\epsilon} \left(1 + \frac{\delta M_\infty}{u_\infty} F_{\bar{\mu}}\right) = \bar{Y}_{K\bar{\mu}} \frac{\delta M_\infty}{u_\infty} P + \alpha^2 [\bar{Y}_{K\bar{\mu}} P_\xi - P_{\bar{\mu}} \bar{Y}_{K\xi} + (r^2 Y_{K\bar{\mu}})_{\bar{\mu}} + F_\xi Y_{K\bar{\mu}} - Y_{K\xi} F_{\bar{\mu}} - \gamma \frac{\delta M_\infty}{u_\infty} (r^2 \bar{Y}_{K\bar{\mu}})_{\bar{\mu}}] \quad (4c)$$

In the limit as $\alpha \rightarrow 0$, the viscous and diffusion terms apparently vanish; this is impossible since the highest derivatives in the equations would be lost, and the boundary conditions could not be satisfied. The cycle times are too short for the diffusion processes to keep up and smooth out the field. Since the diffusion processes are generated at boundaries (in this case the flame), diffusion will become important only near the boundaries. The diffusion "penetration" distance will be small. How small is seen from the fact that to make the highest derivatives of equal importance, a differentiation by $\bar{\mu}$ must raise the order of magnitude of terms by $1/\alpha$ near the boundaries. Therefore, the "high frequency boundary layers" have a thickness of order α .

Another fact becomes important here; no ξ derivatives appear as $\alpha \rightarrow 0$. Therefore, no initial conditions can be satisfied, and the problem apparently becomes independent of initial conditions. It will be assumed that the influence of initial conditions dies out exponentially in a distance of

order α , so that no consideration need be given to them.[†] This is to be regarded a significant result for this compressible jet-mixing problem. However, such behavior is common for parabolic differential equations. For a full discussion of this phenomenon for an incompressible problem, see Lam and Rott.³ This is the great value of the high frequency analysis; no consideration need be given to what happens to the jet or freestream for $s < 0$ in the unsteady state, and the flame zone may be treated independently.

The high frequency solution is obtained by assuming, where $\beta = (\hat{\mu} - \hat{\mu}_f)/\alpha$,

$$P = H_F, o(\hat{\mu}, \xi) + Z_F, o(\beta, \xi) \quad (5a)$$

$$\sigma = S_F, o(\hat{\mu}, \xi) + U_F, o(\beta, \xi) \quad (5b)$$

$$\gamma_K = V_F, o(\hat{\mu}, \xi) + W_F, o(\beta, \xi) \quad (5c)$$

where the F or O subscripts determine to which side of the flame the solution belongs. The functions defined by Eqs. (5) are picked in the following manner: the H , S , and V parts satisfy the full differential equations, Eqs. (4), whereas the Z , U , and W parts only satisfy the homogeneous parts of the differential equations. This is permissible due to the linearity of the problem. Finally, regular series expansions of the form

$$\begin{aligned} H &= \sum_n \alpha^n h^{(n)}(\hat{\mu}, \xi) & Z &= \sum_n \alpha^n z^{(n)}(\beta, \xi) \\ S &= \sum_n \alpha^n s^{(n)}(\hat{\mu}, \xi) & U &= \sum_n \alpha^n u^{(n)}(\beta, \xi) \\ V &= \sum_n \alpha^n v^{(n)}(\hat{\mu}, \xi) & W &= \sum_n \alpha^n w^{(n)}(\beta, \xi) \end{aligned}$$

are assumed. Then it may be shown that a recursive procedure will determine all the unknown functions. This method becomes similar to that developed by Strahle⁴ and suggested by Lam and Rott.³ The boundary conditions, which must also be similarly expanded, are implied in the foregoing statements. The only major difficulty is at the flame position, which is time-dependent. The method is identical to that developed by Strahle⁴ for a leading-edge droplet burning problem.

It should be noted that the introduction of β and the z , u , and w functions brings back the highest derivatives, since $\partial/\partial\hat{\mu} = (1/\alpha)(\partial/\partial\beta)$. This is, in fact, where the rapid high frequency boundary-layer transition takes place in order to satisfy all the boundary conditions. These functions may be shown to have exponential decay away from the boundaries so that the h , s , and v solutions hold in the majority of the field.

The only other keys to the solution lie in an expansion of F as

$$F(\hat{\mu}, \xi) = F[(\alpha\beta + \bar{\mu}_f), \xi] = \sum_{n=0}^{\infty} (\alpha\beta)^n a_n(\xi)$$

and the recognition that inversion of Eq. (3) yields for r

$$\frac{r^2 - \bar{r}_f^2}{2} = \bar{\mu}_f \int_{\bar{\mu}}^{\hat{\mu}} \frac{d\hat{\mu}}{\rho} = \alpha \int_0^{\beta} \frac{d\beta}{\rho}$$

This procedure provides a solution uniformly valid in the entire field free from the blowup of the "infinity" boundary conditions noted by Illingworth.⁵ This represents a refinement of the compressible boundary-layer theory contributed by Illingworth and is a fundamental contribution. This solution is probably, at best, only an asymptotic representation of the actual solution, and the series generated should probably not be interpreted as a convergent one; this is suggested by the incompressible work of Lam and Rott.³

For the solution constructed by the previous method, it may be shown that the burning rate per unit flame length may be evaluated in much the same manner as in Ref. (4),

but with the complication that $r = r(\hat{\mu})$. The result is that, if

$$m_{Ff} = \bar{m}_{Ff} + \epsilon \mathfrak{M}_{Ff} \exp \left[i\omega \left(t + \frac{\delta M_{\infty}}{\bar{u}_{\infty}} \xi \right) \right],$$

$$\frac{\mathfrak{M}_{Ff}}{\bar{m}_{Ff}} = \frac{2R_f}{\bar{r}_f} + \frac{\delta M_{\infty}}{\bar{u}_{\infty}} \left[(\bar{T}_f - \gamma) - \frac{3}{4} \frac{\bar{m}_{Ff}}{\bar{r}_f} q\alpha \right] + O[\alpha^2] \quad (6)$$

where the flame movement, R_f/\bar{r}_f , is given by

$$\frac{R_f}{\bar{r}_f} = \bar{T}_f \frac{\delta M_{\infty}}{\bar{u}_{\infty}} \left[\frac{\bar{T}_f - 1}{2} + \frac{1}{\bar{r}_f^2} \int_0^{\bar{r}_f} r \left(\frac{1}{\bar{T}} - \bar{T} \right) dr \right] + O[\alpha^2] \quad (7)$$

$R_f \rightarrow 0$ as $\bar{r}_f \rightarrow 0$ as expected. Their ratio, however, is bounded. This effect relies on compressibility of the fluid ($M_{\infty} > 0$) and vanishes for an incompressible fluid. The result is rather striking for several reasons. First, to $O[\alpha^2]$ this result requires the compressibility of the fluid. Because of the appearance of δM_{∞} as a multiplicative factor, this result will always follow the pressure (or any other state variable) in the freestream, regardless of the wave type built by superposition. This has previously been called a result due to a "pressure effect."³ Second, the result is bounded at infinite frequency because the flame moves, preventing any very strong mass fraction gradients from appearing near the flame. Third, the infinite frequency limit depends upon the details of the complete flow field only through the flame movement, that is, through R_f . \bar{T}_f appears essentially because the pressure gradient acts upon a fluid of low density near the flame; it is therefore able to "sweep" mass into the flame at a greater rate. Because of the perturbation in $(\rho\mu)$, γ appears and reflects transport property increase due to compression heating. Depending upon the flow field details, R_f/δ can be either positive or negative. It is therefore clear that this limit is either in phase or 180° out of phase with the pressure. However, the fourth point is that this limit is approached from the side which has a component in phase with the pressure, since one component is proportional to $-\delta$ (\bar{m}_{Ff} and q always being positive). The flame movement does not influence this side of approach; all it does is alter the magnitude of the bounded high frequency limit. Fifth, except for the flame movement term, this result is identical in form to a leading-edge droplet burning problem result.⁴ This is because, basically, the physics are the same. Finally, as suggested by Lin's work,⁶ for $M_{\infty} \equiv 0$ this is also a nonlinear result; ϵ does not have to be small. The burning rate is unaffected in the limit of very high frequency for arbitrary amplitude for an incompressible (but still a nonconstant density) fluid. The "acoustic streaming" effect, so important at moderate frequencies as a nonlinear effect, disappears at very high frequencies.

Although this procedure can be continued to gain results for higher orders in α , the algebraic complexity begins to mount rapidly beyond terms of $O[\alpha]$. The results begin to be more and more intimately dependent upon the steady-state flow-field details. Exact numbers can be obtained for \bar{m}_{Ff} and R_f/\bar{r}_f only if exact steady-state flow-field computations are made. As such, these numbers depend upon the initial velocity profile $Y_{o,i}/j$, $Y_{F,c}(0)$, $T_c(0)$, and q ; they are also functions of ξ .

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Influence of Turbulent Boundary Layer on Shock Tube Test Time

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Nomenclature

- τ = real test time at fixed observation point
 τ_i = ideal test time = $(x_s/U_s)(W-1)^{-1}$
 x_s = distance from diaphragm to observation point
 d = hydraulic diameter of shock tube
 U_s = shock front velocity
 M_s = shock Mach number
 ρ = density
 W = ρ_2/ρ_1
 p = pressure
 T = temperature
 a = speed of sound
 ν = kinematic viscosity
 Re = Reynolds number = (ad/ν)

Subscripts

- 1 = initial condition, driven section
 2 = flow region behind shock front
 3 = flow region behind contact front
 4 = initial condition, driver section
 s = condition on shock front
 w = condition on the wall

IN a recent publication, Roshko and Smith¹ used a formula given by Mirels² to compute test times in a shock tube and to compare them with experimental results. The experiments were conducted on the GALCIT 17-in. shock tube using helium as driver gas, and air and argon, respectively, as driven gas.

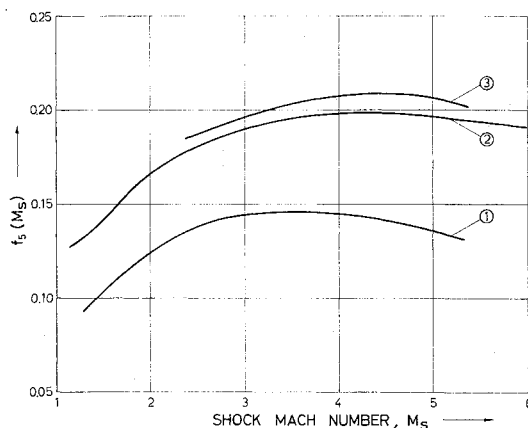


Fig. 1 Function $f_s(M_s)$ for special conditions: ① driver gas air, driven gas air; ② driver gas helium, driven gas argon; ③ driver gas helium, driven gas air; and $T_1 = T_4 = 293^\circ\text{K}$, $T_{2,w} = T_{3,w} = 293^\circ\text{K}$ in each case.

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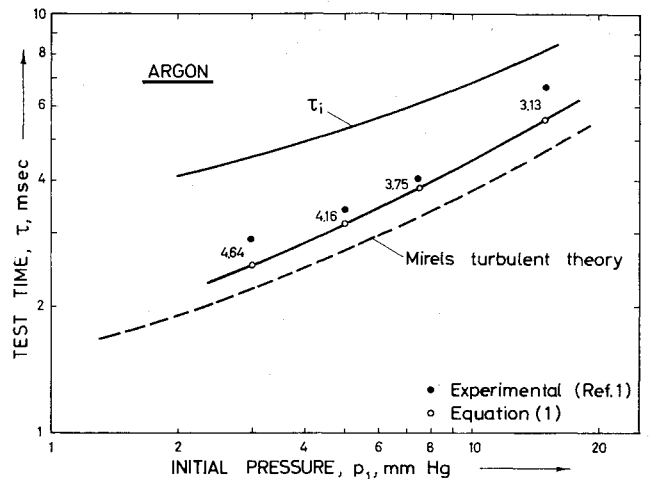


Fig. 2 Test time τ vs initial pressure p_1 ; driver gas helium, driven gas argon; shock Mach number indicated for each experimental point.

In the range of turbulent theory, the agreement between calculated and measured test times is good for air as driven gas, but for argon the computed times are less than the measured ones by about 40%. This discrepancy increases even more if one takes the diaphragm opening time into account.

The purpose of this note is to present an alternative to the comparison carried out by Roshko and Smith. Their experimental results are compared with test times computed from a relation given by Brocher [see Eq. (17) of Ref. 3]

$$\tau/\tau_i = 1 - f_s(M_s) \times Re^{-0.2} \times (x_s/d)^{0.8} \quad (1)$$

In this formula, all the first-order perturbation terms due to the viscous effects are retained. In Fig. 1, the function $f_s(M_s)$ is represented for several conditions of interest. In Figs. 2 and 3, test times computed from Eq. (1) are compared with theoretical predictions based on the relation given by Mirels² and the experimental results from Ref. 1.

Test times calculated from Eq. (1) fit well the experimental data for argon, whereas for air only those for $M_s \leq 3.3$ are more accurate than values obtained from Mirels' formula. Equation (1) is particularly satisfying for small shock Mach numbers M_s . This may be so because Eq. (1) is a linear perturbation formula and may no longer be valid for large M_s , where $\tau/\tau_i \approx 0.5$.

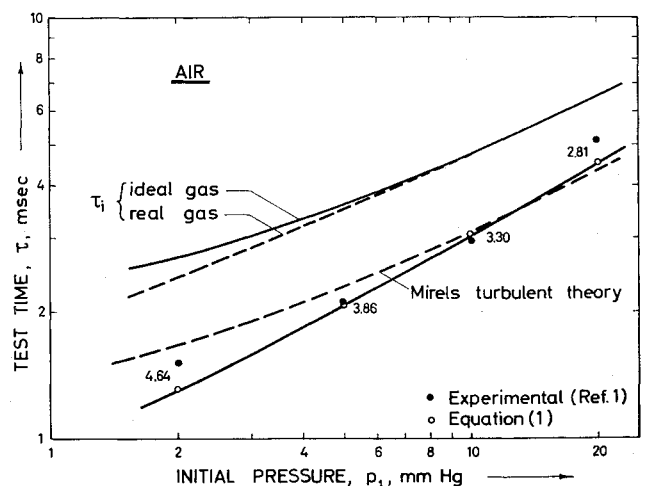


Fig. 3 Test time τ vs initial pressure p_1 , driver gas helium, driven gas air; shock Mach number indicated for each experimental point.